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STRETCHING THE ENDURANCE BOUNDARY OF COMPOSITE MATERIALS: PUSHING THE PERFORMANCE LIMITS OF COMPOSITE STRUCTURES

On the propensity of laminates to delaminate

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Abstract Laminates have a propensity to delaminate; the mathematical plane between adjacent plies offers a preferred path for crack propagation, irrespective of the nature of the stress field that gives rise to the elastic strain energy released. This is because the plane between plies is characterised by a specific fracture surface energy significantly lower than those for internal surfaces that intersect fibres. In the second of his two classical publications on fracture, A.A. Griffith showed how crack rotation in two-dimensional stress fields occurs. This suggests how, in a laminate, pre-existing flaws are able to seek out the plane of lamination; here, crack rotation under the influence of, for example, shear stress is examined in the context of laminates designed for use in aerospace. One physical consequence of Griffith's calculation is the prediction of crack propagation in elastic solids subjected to bidimensional compression with strongly unequal principal stresses. A simple bidimensional compression rig has been devised to investigate this prediction. To obviate the risk of delamination, it will be necessary to move away from anisotropic lay-ups, and further develop three-dimensional weaves and methods for weaving three-dimensional weaves. A method whereby a three-dimensional fibre weave, which has cubic symmetry and no zero-valued shear moduli, might be weaved is outlined.

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Introduction

C.E. Inglis

About 100 years ago, Inglis examined the problem of fracture of ship hulls which, at that time, were made of rivoted steel plate. Along the crack path, he noticed that, as often as not, the rivot holes had been deformed from circular to elliptic shape. This led him to think about the stress magnification around the edge of elliptic holes in plates.

In a plate subjected to a uniaxial tensile stress σ applied in a direction making an angle ϕ with the major axis of the ellipse α_0, Fig. 1, Inglis [1] found that the stress at the surface of, and tangential to, the elliptic hole is

$$\sigma_{\beta\beta} = \sigma\left\{\frac{\sinh 2\alpha_0 + \cos 2\phi - e^{2\alpha_0}\cos 2(\phi - \beta)}{\cosh 2\alpha_0 - \cos 2\beta}\right\}$$

A.A. Griffith

Griffith [2] used this solution to calculate the stress at the surface of the hole when the plate is subjected to a two-dimensional stress field with principal stresses σ_1 and σ_2, respectively making angles ϕ and $\pi/2 - \phi$ with the major axis of the ellipse, Fig. 2.

In Fig. 2, for $\sigma_1 = 0$, the Inglis solution, with the sense of increasing β reversed, gives

$$\sigma_{\beta\beta} = \sigma_2\left\{\frac{\sinh 2\alpha_0 + \cos 2\phi - e^{2\alpha_0}\cos 2(\phi - \beta)}{\cosh 2\alpha_0 - \cos 2\beta}\right\}$$

and, for $\sigma_2 = 0$, it gives

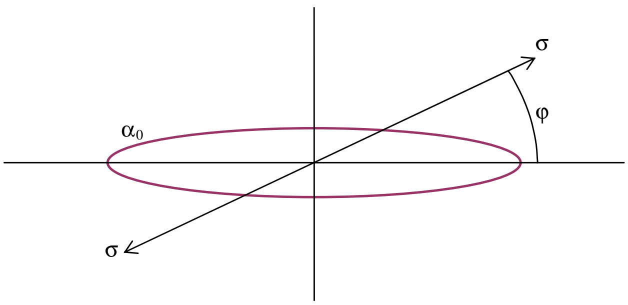

Fig. 1 Elliptic hole in a plate subjected to a uniaxial tensile stress σ

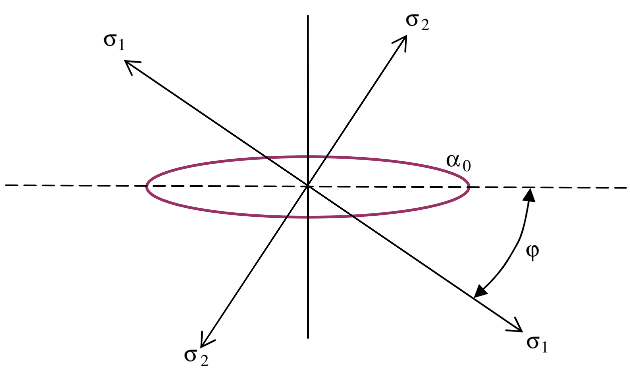

Fig. 2 Elliptic hole in a plate subjected to biaxial tension

$$\sigma_{\beta\beta} = \sigma_1 \left\{ \frac{\sinh 2\alpha_0 + \cos 2\left(\frac{\pi}{2} - \phi\right) - e^{2\alpha_0} \cos 2\left(\frac{\pi}{2} - \phi + \beta\right)}{\cosh 2\alpha_0 - \cos 2(-\beta)} \right\}$$

Superimposing,

$$\sigma_{\beta\beta} = \frac{(\sigma_1 + \sigma_2)\sinh 2\alpha_0 + (\sigma_1 - \sigma_2)\left\{e^{2\alpha_0}\cos 2(\phi - \beta) - \cos 2\phi\right\}}{\cosh 2\alpha_0 - \cos 2\beta}$$

The values of ϕ and β for which $\sigma_{\beta\beta}$ is a maximum are found by differentiation.

Putting $\frac{\partial \sigma_{\beta\beta}}{\partial \beta} = 0$ and taking as solution $\sin 2\beta = A\alpha_0 + 0(\alpha_0^2)$, Griffith found that $\sigma_{\beta\beta}$ is a maximum at two pairs of points.

If $\phi = 0$ or $\frac{\pi}{2}$, these points are at the ends of the major and minor axes, respectively.

For all other values of ϕ, both pairs of points are close to, but not at the ends of the axes, as sketched in Fig. 3

The two extremal values of $\sigma_{\beta\beta}$ are always of opposite sign.

To find where, on the crack surface, $\sigma_{\beta\beta}$ takes its extremal values, it is necessary to evaluate

$$\frac{\partial \sigma_{\beta\beta}}{\partial \phi} = 0$$

This leads to two conditions for fracture:

Taking tensile stress as +ve and $\sigma_2 > \sigma_1$,

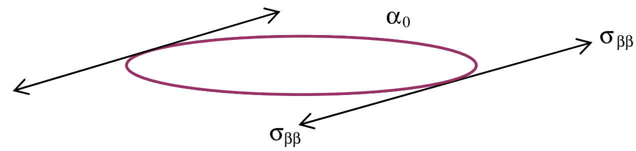

Fig. 3 Illustrating the off-major axis locations where the tensile stress, tangential to the surface of a crack in a plate subjected to a two-dimensional stress field, is highest

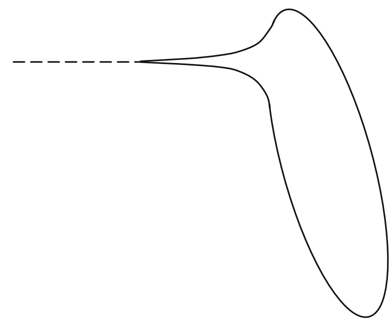

Fig. 4 Schematic drawing of an internal elliptic hole located totally within a laminate. The broken line emanating from the starter crack, denotes the orientation of the plane of lamination

(i) If $3\sigma_2 + \sigma_1 > 0$, fracture occurs when $\sigma_2 > K$, where K is the strength in uniaxial tension, and $\phi = 0$, i.e. the fracture surface is perpendicular to σ_2.

(ii) If $3\sigma_2 + \sigma_1 < 0$, fracture occurs when $(\sigma_2 - \sigma_1)^2 + 8K(\sigma_2 + \sigma_1)^2 = 0$ and $\cos 2\phi = -\frac{1}{2}\frac{(\sigma_2 - \sigma_1)}{(\sigma_2 + \sigma_1)}$ and crack growth proceeds from near, but not at, the end(s) of the major axis of the pre-existing flaw and is on a plane inclined to the directions of principal stress as illustrated in Fig. 4.

Note that, in uniaxial compression, $\sigma_2 = 0$, $\sigma_1 = -\sigma$, equation (ii) applies and fracture is predicted when $\sigma = -8K$; that is the uniaxial compressive strength of an elastic solid is expected to be eight times its uniaxial tensile strength, an empirical fact well known to the ancient Greeks and Romans. Also note that, when shear is introduced, the crack rotates. In a layered material such as a laminate, this rotation enables the starter crack to seek out the inter-ply plane of weakness as illustrated in Fig. 4.

E. Orowan

Orowan [3] was the first to draw attention to the graphical representation of equations (i) and (ii) sketched in Fig. 5. While (ii) is the equation to a parabola, open in the compression–compression quadrant, (i) is the equation to a tangent to this parabola. Fracture is expected when the state of stress moves from inside to outside the fracture locus.

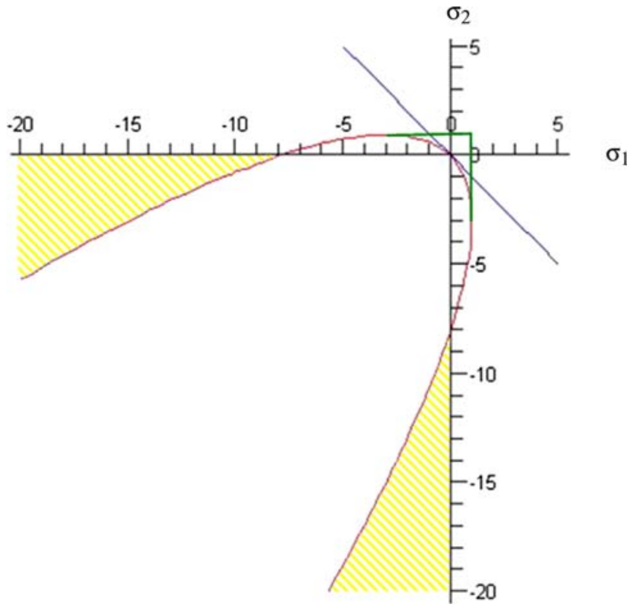

Fig. 5 Orowan's fracture locus for elastic solids. To give the plot order of magnitude significance, the units of stress are given in kilobars

Of special interest in geophysics are the shaded areas in Fig. 5. Here, fracture is predicted when both principal stresses are compressive and strongly unequal. This non-obvious prediction has been used by Orowan to develop models for mechanical behaviour of the upper mantle; the Earth continues to cool down and radial shrinkage associated with this cooling creates circumferential shrinkage in the upper mantle which, because the mantle is a thin shell, manifests itself as two-dimensional compression in the horizontal plane.

In aeronautics, propulsion works against drag and lift works against payload to create an overall stress field that is two-dimensional. Of particular interest in materials selection for aerospace applications is the fact that the overall stress field experienced by airframes includes bidimensional compression; the possibility that the principal stresses can become strongly unequal (during "heavy landing", for example) warrants investigation.

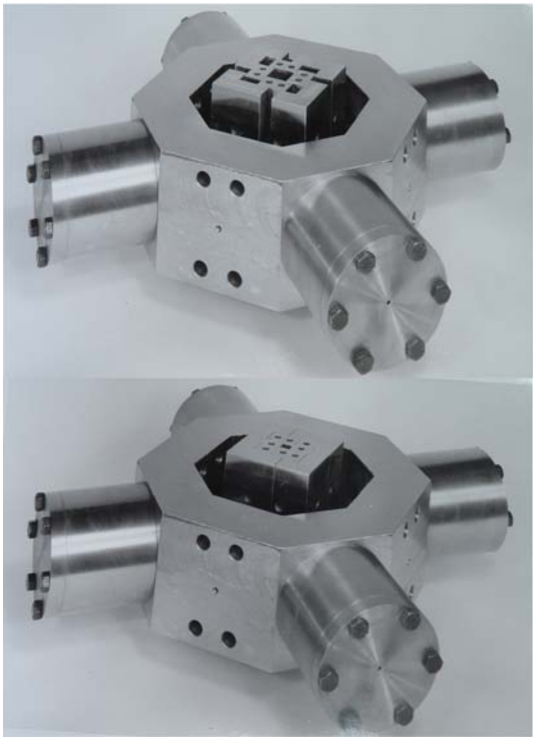

Fig. 7 A hydraulically driven bidimensional compression device. When fully open (upper photograph), the confined cell measures 20 mm × 20 mm × 200 mm. When fully closed (lower photograph), it measures 10 mm × 10 mm × 200 mm

Experimental bidimensional compression

Figure 6 is a model of a bidimensional compression device [4]. The anvils are "G" shaped, and opposing pairs of anvils can be advanced independently of each other. A hydraulically driven working version of the invention is shown in Fig. 7.

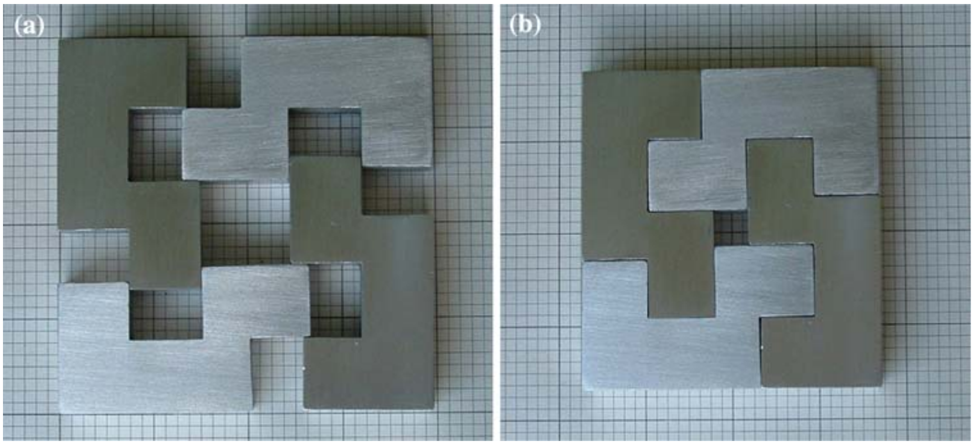

Fig. 6 Interlocking "G" shaped anvils of a bidimensional compression device [4]. (**a**) anvils open, (**b**) anvils closed

Overcoming the problem of delamination

Delamination heralds the start of mechanical failure of laminates. When subjected to repeated deformation by bending, areas of delamination appear, thereby rendering the laminate more compliant so that, for the same applied forces, it bends more, causing fibres to become sufficiently tensioned that fibre fracture commences, rendering the laminate even more compliant so that it bends even more easily and broken fibre lengths pull out. Summarising, delamination occurs first, is followed by the onset of fibre fracture, and then by fibre pull-out.

To avoid the problem of delamination, a three-dimensional fibre array is required. A woven version of one such array, specifically one that imparts high shear modulus in all three dimensions, is illustrated in Fig. 8, [5]. To see how this weave might be manufactured, in Fig. 9, consider a "cube body diagonals" array of helical fibres. Compact the array three-dimensionally, until the helices touch, and arrange for the helices to cross over at their points of contact. In practice, this might be achieved by simultaneously withdrawing (along) and rotating four families of fibres (about) the four body diagonals of a cube and, when they touch, disconnecting/reconnecting individual fibres in order to create the required fibre cross-overs. When the fibres are pulled tight, the array becomes a cubic symmetry 3D weave of just four families of helical fibres wound, not on right circular cylinders as generators, but on right pyramids of equilateral triangular cross section. All fibre lengths are parallel to cube face diagonals. In Fig. 8, note that the fibre cross-overs hold the array together; this is shown in detail in Fig. 10.

The specific fracture surface energy γ for composites based on the three-dimensional fibre weave illustrated in Fig. 8 is not expected to be isotropic. Rather, it is anticipated that γ will vary with the number of fibres intersected,

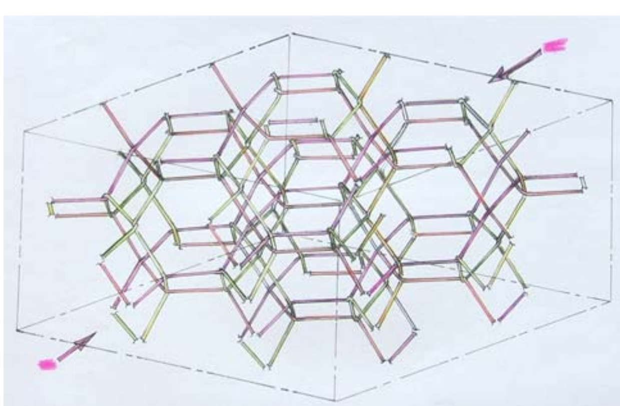

Fig. 8 A cubic symmetry three-dimensional weave, of four families of cube "body diagonal" fibres, that has no zero-valued shear moduli [5]

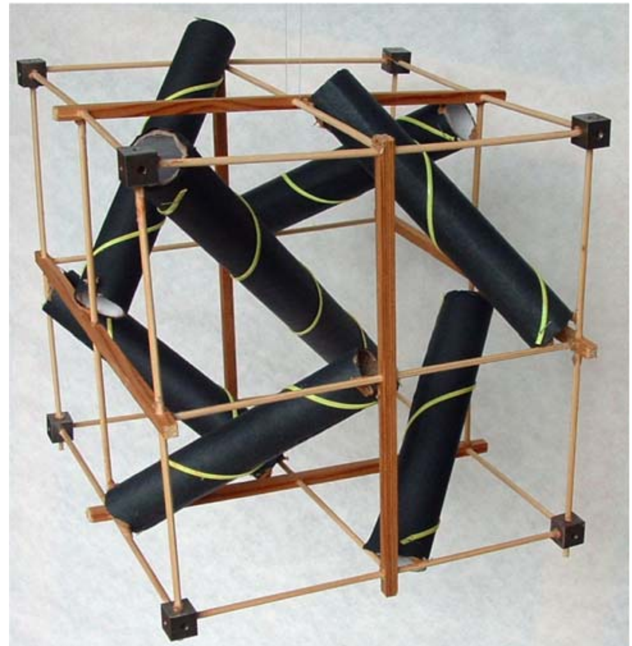

Fig. 9 A cube "body diagonals" array of helically wound fibres

i.e. γ will be orientation dependant. Over a solid angle about any chosen direction, there will be a range of orientations of planes for which the number of fibre segments intersected by unit area is constant. For example, over a solid angle of, say, 45° about a cube edge (of the mother cube) or about a face diagonal direction, it is 4, compared with 6 about body diagonal direction. For this reason, a polar diagram of γ will be similar in appearance to the

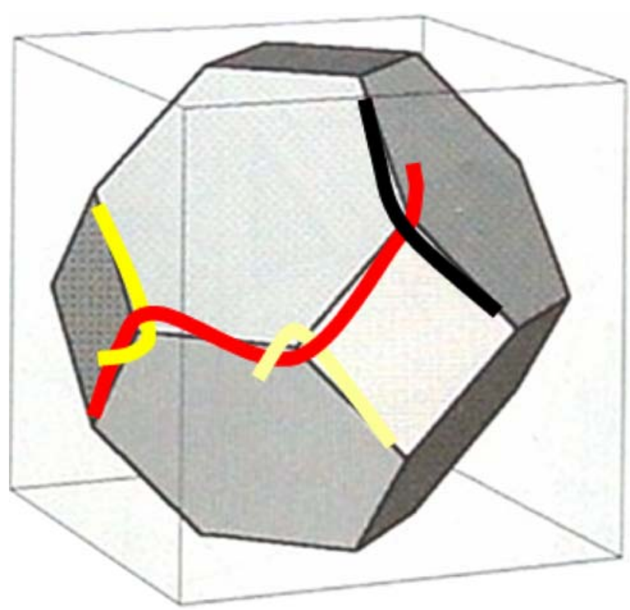

Fig. 10 The fibre cross-overs that hold together the fibre weave illustrated in Fig. 9

surface of a raspberry in that it is constructed from spheres drawn through the origin. In the context of representation of anisotropy of surface energy of crystals, Frank [6] pointed out that a geometrically simpler representation is the γ^{-1}—polyhedron obtained by inverting these spheres through the origin when they become planes.

The weave shown in Fig. 8 constitutes a lattice. In Fig. 10, it is evident that the centres of mass (and of electron density) of this lattice are at the fibre cross-overs, suggesting that the corresponding reciprocal lattice might be that for centres of electron density located at the corners of space-filling equilateral truncated octahedra, i.e. that for simple cubic crystals. It is of some interest to consider the likely modes of wave propagation by such a periodic structure (J.C. Gill (2006) Private communication) and, in particular, the propagation of electromagnetic waves when the fibre material, but not the matrix material, is a good conductor. Laminates with fibre volume fraction $\eta > 1/2$, modelled as alternate slabs of conducting and insulating material, are a special case in that an electromagnetic wave can have **E** perpendicular to, and **B** parallel to, all conducting surfaces. This allows the boundary conditions at the surfaces to be satisfied even if the wavelength is much greater than the separation of the slabs; the wave can then propagate, without loss if the surface conductivity is infinite. However, in three-dimensional composites woven with fibres that are good conductors, the boundary conditions can be satisfied only if the directions of **E** and **B** are able to reverse in distances of the same order of magnitude as the fibre separation (d). That limits propagation to wavelengths shorter than some critical value of the order of 2d; with shorter wavelengths, the wave would propagate, again without loss, if the conductivity is infinite.

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